

# SPTCC Summer School on Concurrent Computing

June 3-7, 2017

Simple exercises related to Michel Raynal's lectures

## 1 Exercise 1: on the kFK universal construction

Slides 50-60 presented a very simple universal construction suited to the computation model  $\mathcal{CARW}[LL/SC]$ . This construction uses an internal procedure `apply()`, which is based on a “repeat twice” loop statement. Let us redefine this operation where “repeat twice” is replaced by a “repeat until” statement as follows:

```
internal procedure apply() is
  repeat
     $ls \leftarrow STATE.LL()$ ;
     $pairs \leftarrow BOARD.collect()$ ;
    for  $\ell \in \{1, 2, \dots, n\}$  do
      if ( $pairs[\ell].sn = ls.sn[\ell] + 1$ ) then
         $\langle ls.value, r \rangle \leftarrow \delta(ls.value, pairs[\ell].op)$ ;
         $ls.res[\ell] \leftarrow r$ ;
         $ls.sn[\ell] \leftarrow pairs[\ell].sn$ 
      end if
    end for
  until  $STATE.SC(ls)$  end repeat.
```

Is this modification correct? If it is correct provide a proof of it. If it is incorrect, provide a counter-example.

## 2 Exercise 2: on operations on memory locations

Let us consider the model  $\mathcal{CARW}[\emptyset]$  enriched with the following atomic hardware-provided operations. Hence (as the read and write operations) these operations can access any memory location, or the very same location. Let  $X$  denote a memory location, and  $\alpha$  an integer greater than 1.

- $X.multiply(\alpha)$  multiplies by  $\alpha$  the value in  $X$ . (Hence if  $X = x$  when  $X.multiply(\alpha)$  is invoked we have  $X = \alpha \times x$  when it returns.)
- $X.decrement()$  decrements by 1 the value in  $X$ . (Hence if  $X = x$  when  $X.decrement()$  is invoked we have  $X = x - 1$  when it returns.)

Show that in the system  $\mathcal{CARW}[\emptyset]$  enriched with  $multiply(\alpha)$  and  $decrement()$  (in addition to  $read()$ ), consensus can be solved for ANY number of processes. To this end design (and prove correct) in this computing model, a binary consensus algorithm which works for any number of processes.

If time permits (... much more difficult ...) try to show that binary consensus for 2 processes is impossible in  $\mathcal{CARW}[\emptyset]$  enriched with only one of the operations  $decrement()$  or  $multiply(\alpha)$ . (Actually,  $\mathcal{CARW}[\emptyset]$  enriched with only one of these operations has consensus number 1.)