

SPTCC Summer School on Concurrent Computing

June 3-7, 2017

Simple exercises related to Michel Raynal's lectures

1 Exercise 1: on the kFK universal construction

Slides 50-60 presented a very simple universal construction suited to the computation model $\mathcal{CARW}[LL/SC]$. This construction uses an internal procedure `apply()`, which is based on a “repeat twice” loop statement. Let us redefine this operation where “repeat twice” is replaced by a “repeat until” statement as follows:

```
internal procedure apply() is
  repeat
     $ls \leftarrow STATE.LL()$ ;
     $pairs \leftarrow BOARD.collect()$ ;
    for  $\ell \in \{1, 2, \dots, n\}$  do
      if ( $pairs[\ell].sn = ls.sn[\ell] + 1$ ) then
         $\langle ls.value, r \rangle \leftarrow \delta(ls.value, pairs[\ell].op)$ ;
         $ls.res[\ell] \leftarrow r$ ;
         $ls.sn[\ell] \leftarrow pairs[\ell].sn$ 
      end if
    end for
  until  $STATE.SC(ls)$  end repeat.
```

Is this modification correct? If it is correct provide a proof of it. If it is incorrect, provide a counter-example.

2 Exercise 2: on operations on memory locations

Let us consider the model $\mathcal{CARW}[\emptyset]$ enriched with the following atomic hardware-provided operations. Hence (as the read and write operations) these operations can access any memory location, or the very same location. Let X denote a memory location, and α an integer greater than 1.

- $X.multiply(\alpha)$ multiplies by α the value in X . (Hence if $X = x$ when $X.multiply(\alpha)$ is invoked we have $X = \alpha \times x$ when it returns.)
- $X.decrement()$ decrements by 1 the value in X . (Hence if $X = x$ when $X.decrement()$ is invoked we have $X = x - 1$ when it returns.)

Show that in the system $\mathcal{CARW}[\emptyset]$ enriched with $multiply(\alpha)$ and $decrement()$ (in addition to $read()$), consensus can be solved for ANY number of processes. To this end design (and prove correct) in this computing model, a binary consensus algorithm which works for any number of processes.

If time permits (... much more difficult ...) try to show that binary consensus for 2 processes is impossible in $\mathcal{CARW}[\emptyset]$ enriched with only one of the operations $decrement()$ or $multiply(\alpha)$. (Actually, $\mathcal{CARW}[\emptyset]$ enriched with only one of these operations has consensus number 1.)