

- 7.1. Show that if you make splay operation using only zig every time, then there is a sequence of  $n$  operations that take longer than  $n \log n$  time.
- 7.2. Let the splay tree contain numbers from 1 to  $n$ . We will do  $m$  operations: `splay(1)`, `splay(n)`, `splay(1)`, `splay(n)`, ... What will be the total time of all operations?
- 7.3. In a splay tree of size  $n$ , you perform many operations on a small subset of  $k$  elements. What will the tree look like? How long will the operations work?
- 7.4. Let the AVL tree contain numbers from 1 to  $n$ . Show that if you do the find operation in order for all numbers from 1 to  $n$ , then the total running time will be more than  $O(n)$ .
- 7.5. Let the splay tree contain numbers from 1 to  $n$ .  $M$  queries are made to the tree, the total number of queries to  $i$ -th element is  $p_i$ . Show that the total time complexity of all queries is  $O(m + \sum p_i \log \frac{m}{p_i})$  (hint: you need to choose the correct  $w(v)$  so that  $r(x)$  is as need to).
- 7.6. Show how to do the `split` operation in the splay tree. How long will will it work? (do not forget to calculate the change in potential).
- 7.7. Show how to do the `merge` operation in the splay tree. How long will will it work? (do not forget to calculate the change in potential).
- 7.8. Show how to build a splay tree from a given sorted array in  $O(n)$  amortized time. (do not forget to calculate the change in potential).