3.1. Consider a path from v to u in which the weight of the maximum edge is minimal. Show that it is possible to construct a path with the same maximum along the edges of the minimal spanning tree.

3.2. Consider an arbitrary cycle in the graph. Show that the maximum edge in this cycle does not lie in the minimum spanning tree.

3.3. You need to transfer n files from one computer to another. Each file is a bit vector of size m. The file can be sent either simply as a sequence of bits, or as a difference with another file already sent earlier. In the first case, you will need to transfer m bits, in the second - $A + B \times d$ bits, where d is the number of different bits, A and B are some constants. Find the minimum number of bits you need to transfer.

3.4. There are n cities. You can connect two cities by a road by spending $A \times len$ money, where len is the length of the road, or you can build an airport in the city by spending B money. Find the minimum amount of money to connect all the cities (so that from each city all other cities are reachable using roads and airplanes).

3.5. Given an unweighted directed graph. For each edge check if there is a shortest path from s to t that passes through it. Time $O(m)$.

3.6. Given a directed graph, edge weights equal to 0 or 1. Find the shortest path. Time $O(m)$.

3.7. Given a directed graph. You can reverse (change direction) of some edges. Find the minimum number of edges you need to reverse to make a path from s to t. Time $O(m)$.

3.8. Given a directed graph. You can reverse at most one edge, find the minimal possible distance from s to t you can get. Time $O(m)$.

3.9. Given a directed graph with letters on each edge. Consider all the shortest (by the number of edges) paths from the vertex s to the vertex t. For each path, write out a string of the corresponding letters. Find the path for which this string is lexicographically minimal. Time $O(m)$.