

- 1.1. An acyclic directed graph is given. Cover all its vertices with the minimum number of vertex-disjoint paths.
- 1.2. The maximum independent set problem. Given a bipartite graph. Choose the maximum number of vertices so that there is no edge connecting the selected vertices.
- 1.3. Given a bipartite graph and a maximum matching in it. Check if this is the only maximum matching in  $O(n + m)$ .
- 1.4. Prove that a  $d$ -regular bipartite graph (that is, a graph in which the degrees of all vertices are equal to  $d$ ) has a perfect matching, and that a  $d$ -regular graph can be decomposed into  $d$  disjoint perfect matchings.
- 1.5. The defect of the set of vertices of the left part in the graph is  $def(A) = |N(A)| - |A|$ . Find a set with minimal defect in a bipartite graph.
- 1.6. There is a board  $n \times n$ , some cells of which have been removed. Put the maximum number of dominoes  $2 \times 1$  on the board.
- 1.7. There is a board  $n \times n$ , some cells of which have been removed. Place the maximum number of chess knights so that they do not attack each other.
- 1.8. There is a board  $n \times n$ , some cells of which have been removed. Place the maximum number of rooks so that they do not attack each other (rooks attack through empty squares).
- 1.9. There is a board  $n \times n$ , each cell is either free or with a wall. Place the maximum number of rooks so that they do not attack each other (rooks do not attack through walls).