Day 5: Problem Analysis

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Preliminaries
Problem A
Problem B
Problem C
Problem D
Problem E
Problem F
Problem G
Problem H
Problem I
Problem J
Problem K

Preliminaries

- Various hard problems from different contests

A–C	←	Guangzhou 2003
D–G	←	Shanghai 2004
H	←	Phuket 2009
I, J	←	MCPC 2008
K	←	Dhaka 2002

- Time limits for most problems were tightened

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Problem A. Elevator Stopping Plan

Statement

- People want to get to their floors as soon as possible
- There is an elevator
  - speed: 4 seconds / floor
  - unloading: 10 seconds
  - by foot: 20 seconds / floor
- Given needed floors, construct the optimal stopping points

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Problem A. Observations

Troubles

- If you think you need to stop only on input floors, you are in trouble
  - 8 2 4 6 8 10 12 14 16
  - Optimal answer 80: 3 8 13 16
  - You would find 90: 4 8 12 14 16

Problem A. Solution

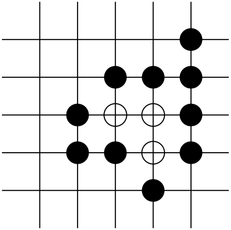
Dynamic programming

- $A(f, t)$ – what is the maximum time for people aiming at floors $\leq t$, if we stop at f floors and t is the last one
- Convenient thing: from f and t you can restore the elevator stopping time at floor t
- Recomputation:
  - consider all next stopping floors t'
  - find the maximum for people from (t, t')
  - update $A(f + 1, t')$
- At the end: count everyone $> t$ as well

Problem B. New Go Game

Statement

- A field with stones
- Known only the numbers of stones for rows, columns, diagonals
- Output the number of enclosed intersections



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Problem B. Solution

- Brute force: for each cell try 1 then 0
- Evident heuristics:
  - track how many zeros and ones remain at each row, column, diagonal
  - if 1 or 0 are impossible, don't try them
- That's still TLE
- Non-evident heuristics (both AC):
  - ITMO 1: choose a cell from row/column/diagonal with smallest number of choices (binomal quotient)
  - my: first, test if there are cells with at most one way to fill

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Problem B. Reflections

- We don't have proofs that our solutions really work fast for every possible test case
  - Probably, they can be challenged
  - Probably, the tests are weak
- I am unaware of a "normal" solution with predictable behavior

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Problem C. Outernet

Statement

- Description of interaction protocol for application proxies
- Given routing table and requests, output what should the application proxy say

Solution

- Implement the protocol carefully
  - Maintain computer names for input and output ports
  - Close sessions timely
  - Don't unescape escape characters
  - ...

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Problem D. Amphiphilic Carbon Molecules

Statement

- Given black or white points on a plane
- Separate them by a line so that number of black points on one side plus the number of white points on other side is maximum possible

Problem D. Solution

Idea

The line always passes through at least two points!

Implementation

- Fix one point at time
- Sort other points around it
- Linear scan for maximum answer
- Complexity: $O(N^2 \log N)$

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Problem E. Different Digits

Statement

- Given an integer X $(1 \le X \le 65535)$
- Find a minimum Y such that $X \cdot Y$ contains minimum possible number of different digits

Solution idea

- The minimum number of different digits is always 1 or 2

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Problem E. Solution

Check if one digit (D) is possible

- Track remainders of D, DD, DDD mod X
  - either hits 0 or enters a loop

For two digits $D_1 < D_2$

- Graph: vertices = remainders mod X
- Edges: appending digits
- BFS: first use D_1, then use D_2
  - first hit of 0 is lex-smallest
  - don't use $D_1 = 0$ in the beginning

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Problem F. The Floor Bricks

Statement

- Given several Tetris-like pieces (fit in 3×3)
- Each has a price
- Fill a given region with these pieces in a cheapest way

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Problem F. The Floor Bricks

Statement

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- Each has a price
- Fill a given region with these pieces in a cheapest way

Key insight

- Max region height: 5 (look up the statement)
- Dynamic programming is possible

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Problem F. Solution

- Track the current three columns of region
  - at most $3 \times 5 = 15$ bits
- Try fitting all figures and advancing
- Convenient bit representation (for me):
  - bit packing for current region:

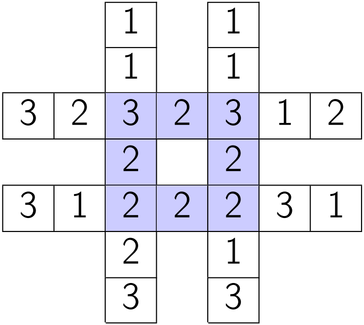
14	9	4
13	8	3
12	7	2
11	6	1
10	5	0

  - advantage: after insertion of a figure, the integer value increases
- When the first column is filled, advance one step right

Problem G. The Rotation Game

Statement

- Make colored cells contain equal numbers
- Operations: vertical/horizontal band rotation



Problem G. Solution

Key idea

- Total states: $\binom{24}{8} \cdot \binom{16}{8} = 9465511770$
- Total states with two types merged: $\binom{24}{8} = 735471$
- Test values to be in the middle separately

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Problem G. Solution

Key idea

- Total states: $\binom{24}{8} \cdot \binom{16}{8} = 9465511770$
- Total states with two types merged: $\binom{24}{8} = 735471$
- Test values to be in the middle separately

Solution

- Construct the graph once
- For each test case, check all values
- Lex smallest shortest path using the graph

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Problem H. Hexagonal Sticks

Statement

- Given hex field and sticks connecting centers of adjacent cells
- Some obstacles on the field
- Make a hexagon from sticks in min moves

Solution

- Only 6-stick hexagons are possible
- Shortest paths in stick location graphs
- For each hexagon center, find minimum matching, the best minimum is the answer

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Problem I. Hex Tile Equations

Statement

- At most 14 hexagon cells with digits and operators
- Find a hamiltonian path which generates a valid expression

Solution

- Not many valid paths, roughly at most: $\min(6 \cdot 2^4 \cdot 5^8, 3 \cdot 2^3 \cdot 5^9) = 3.75 \cdot 10^7$
- Check them all and find the answer

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Problem J. Line & Circle Maze

Statement

- Intersecting line segments and circles
- Find longest non-infinite path

Solution

- Find all intersection points
- Join them in a graph
- Floyd-Warshall
- Find maximum non-infinite number

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Problem J. Geometry Concerns

- Segment vs Segment
  - $S_x^1 + t \cdot (T_x^1 - S_x^1) = S_x^2 + u \cdot (T_x^2 - S_x^2)$
  - $S_y^1 + t \cdot (T_y^1 - S_y^1) = S_y^2 + u \cdot (T_y^2 - S_y^2)$
  - Solve for t and u, check for $\in [0; 1]$
- Segment vs Circle
  - S, T: endpoints, C: circle center, r: radius
  - $(S + t \cdot (T - S) - C)^2 = r^2$
  - Solve quadratic equation on t
  - Check solutions for $\in [0; 1]$
- Circle vs Circle
  - Subtract circle equations, get a line equation
  - Call previous subroutine (no $[0; 1]$ check!)

Problem K. Enigmatic Travel

Statement

- Full graph with N vertices
- All journeys with length $\leq N$ considered
  - each journey is equiprobable
- Find expected lengths
  - in general case
  - if a journey is a simple path
  - if a journey is a simple cycle

Problem K. Solution

Expectation: $E = \frac{\sum_L L \cdot f(L)}{\sum_L f(L)}$,
where $f(L)$ – number of journeys of length L

1. General case:
   - For length $L \geq 1$: $(N-1)^{L-1}$ journeys
     - each move except for the last one is to a vertex different to the current one
2. Simple path case:
   - For length $L \geq 1$: $\frac{(N-2)!}{(N-L-1)!}$
     - choose a vertex subset to use
     - choose an order of visiting
3. Simple cycle case:
   - For length $L \geq 3$: $\frac{(N-1)!}{(N-L)!}$