ACM ICPC 2015–2016
Northeastern European Regional Contest
Problems Review

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December 6, 2015

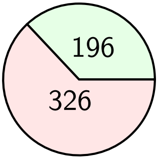
Problems summary

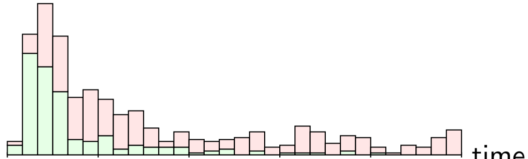
- Recap: 224 teams, 12 problems, 5 hours,
- 20-th NEERC — Jubilee
- This review assumes the knowledge of the problem statements (published separately on http://neerc.ifmo.ru/ web site)
- Summary table on the next slide lists problem name and stats
  - author — author of the original idea
  - acc — number of teams that had solved the problem (gray bar denotes a fraction of the teams that solved the problem)
  - runs — number of total attempts
  - succ — overall successful attempts rate (percent of accepted submissions to total, also shown as a bar)

Problems summary (2)

problem name	author	acc/runs	succ
Adjustment Office	Vitaliy Aksenov	196 /522	37%
Binary vs Decimal	Mikhail Tikhomirov	12 /83	14%
Cactus Jubilee	Mikhail Tikhomirov	14 /28	50%
Distance on Triangulation	Gennady Korotkevich	9 /67	13%
Easy Problemset	Andrey Lopatin	208 /281	74%
Froggy Ford	Georgiy Korneev	101 /602	16%
Generators	Elena Andreeva	153 /533	28%
Hypercube	Oleg Khristenko	3 /15	20%
Iceberg Orders	Egor Kulikov	0 /2	0%
Jump	Maxim Buzdalov	52 /577	9%
King's Inspection	Mikhail Dvorkin	24 /253	9%
Landscape Improved	Georgiy Korneev	56 /213	26%

Problem A. Adjustment Office





	Java	C++	Total
Accepted	17	179	196
Rejected	53	273	326
Total	70	452	522

solution	team	att	time	size	lang
Fastest	SPbAU 1	1	7	1,051	C++
Shortest	NU 14	2	41	529	C++
Max atts.	UrSU 2	10	130	1,650	Java

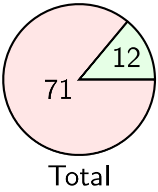
Problem A. Adjustment Office (1)

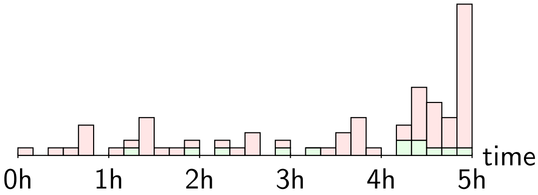
- Recap: $n \times n$ grid, each cell (x, y) has value $x + y$
- Initial sum at row or column k is equal to $nk + \frac{n(n+1)}{2}$
- Maintain two pieces of data for rows and columns:
  - The set of all zeroed out rows $\mathcal{S}_\mathrm{r}$ and columns $\mathcal{S}_\mathrm{c}$, initially both sets are empty
  - Two sums $s_{\mathrm{r},\mathrm{c}} = \sum_{i \in \{1 \ldots n\} \setminus \mathcal{S}_{\mathrm{r},\mathrm{c}}} i$, initially both sums are $\frac{n(n+1)}{2}$
- The result of row query "`R` r" is equal to:

$$\begin{cases} \left(n - |\mathcal{S}_\mathrm{c}|\right) r + s_\mathrm{c} & \text{if } r \notin \mathcal{S}_\mathrm{r} \\ 0 & \text{otherwise} \end{cases}$$

- If $r \notin \mathcal{S}_\mathrm{r}$, then $\mathcal{S}_\mathrm{r} \leftarrow \mathcal{S}_\mathrm{r} \cup \{r\}$ and $s_\mathrm{r} \leftarrow s_\mathrm{r} - r$
- Similarly for column queries

Problem B. Binary vs Decimal





	Java	C++	Total
Accepted	4	8	12
Rejected	4	67	71
Total	8	75	83

solution	team	att	time	size	lang
Fastest	SPb SU 1	2	73	1,914	Java
Shortest	ITMO 1	1	112	1,193	Java
Max atts.	MAI	5	269	256,031	C++

Problem B. Binary vs Decimal (1)

- It is easy to prove that ${10^k}_2$ has ${2^k}_2$ as a suffix, for example:

decimal	binary
1_{10}	1_2
10_{10}	1010_2
100_{10}	1100100_2
1000_{10}	1111101000_2

- Let $\mathcal{C}_k$ be a set of all numbers less than 10^k, whose decimal representation is equal to its k last binary digits
- Let $\mathcal{C}_k = \mathcal{A}_k \cup \mathcal{B}_k$, where all $x \in \mathcal{A}_k$ have k-th (counting from zero) digit of 0 and all $x \in \mathcal{B}_k$ have k-th digit of 1
- $\mathcal{B}_k$ is a set of *bindecimal* numbers of length k — one we want

k	$\mathcal{A}_k$	$\mathcal{B}_k$
0	$\{0\}$	$\{\}$
1	$\{0\}$	$\{1\}$
2	$\{0, 1\}$	$\{10, 11\}$
3	$\{0, 1, 10, 11\}$	$\{100, 101, 110, 111\}$
4	$\{0, 1, 100, 101\}$	$\{1000, 1001, 1100, 1101\}$

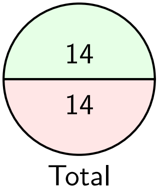
Problem B. Binary vs Decimal (2)

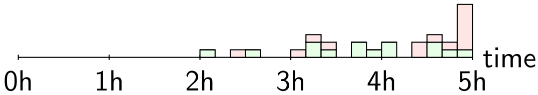
- Define a recursive rule to get $\mathcal{C}_k$ from $\mathcal{C}_{k-1}$

$$
\begin{aligned}
\mathcal{A}_k &= \{x && \mid x \in \mathcal{C}_{k-1} \text{ and } k\text{-th bit of } x \text{ is zero}\} \\
\mathcal{B}_k &= \{x + 10^k && \mid x \in \mathcal{C}_{k-1} \text{ and } k\text{-th bit of } x \text{ is zero}\} \\
\mathcal{C}_k &= \mathcal{A}_k \cup \mathcal{B}_k
\end{aligned}
$$

- Keep $\mathcal{C}_k$ as an ordered list, so when new $\mathcal{B}_k$ is computed as defined above, the bindecimal numbers in $\mathcal{B}_k$ are produced in ascending order; count them; stop when n-th bindecimal number is found
- Note, that the max answer for $n = 10\,000$ is around 10^{161}, so it will not fit into any standard data types; need long arithmetics to implement it

Problem C. Cactus Jubilee



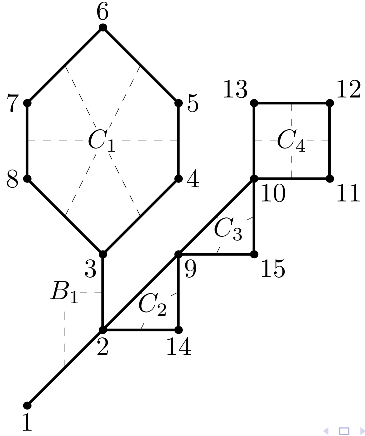


	Java	C++	Total
Accepted	0	14	14
Rejected	0	14	14
Total	0	28	28

solution	team	att	time	size	lang
Fastest	Saratov SU 4	1	124	3,673	C++
Shortest	Ural FU 1	2	151	2,735	C++
Max atts.	MIPT 2	4	279	3,487	C++

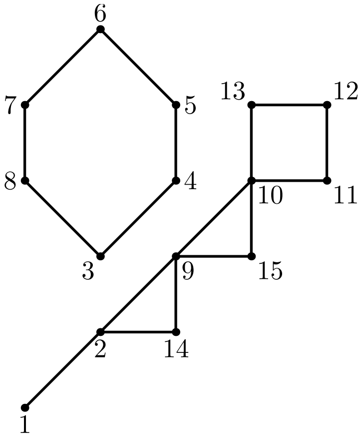
Problem C. Cactus Jubilee (1)

- Depth-first search (DFS) of the cactus to split the edges of the cactus into disjoint sets of *bridge-trees* B_i and *cycles* C_i
  - Each back edge found during DFS signals a cycle
  - Compute size of each B_i and a number of non-adjacent pairs of edges during the first DFS; do a second DFS to push it to all adjacent cycles



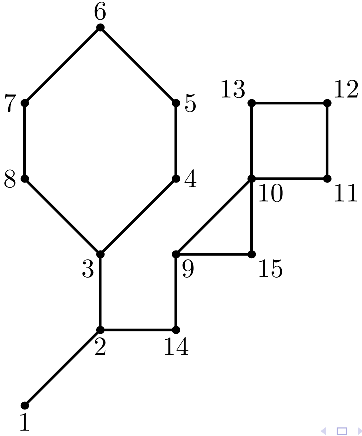
Problem C. Cactus Jubilee (2)

- When an edge from a bridge-tree B_i is removed, cactus splits into two connected components
  - any pair of vertices from these two components can be reconnected to get a cactus
  - number of ways can be counted during initial DFS

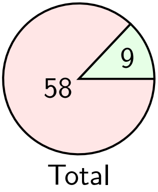


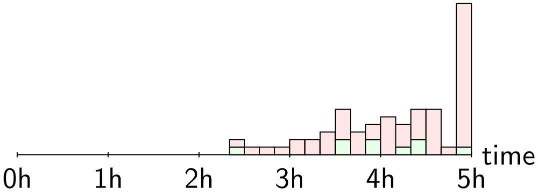
Problem C. Cactus Jubilee (3)

- When an edge from a cycle C_i is removed, cactus is still connected; bridge-trees adjacent to cycle merge
  - any pair of non-adjacent vertices from the same bridge-tree can be connected to get a cactus
  - Scan all cycles to figure out the the number of ways to add an edge for each cycle broken; multiple by the cycle size



Problem D. Distance on Triangulation



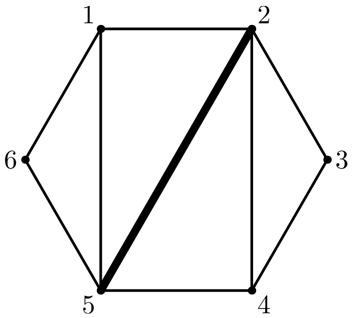


	Java	C++	Total
Accepted	0	9	9
Rejected	2	56	58
Total	2	65	67

solution	team	att	time	size	lang
Fastest	NNSU	1	145	4,602	C++
Shortest	SPbAU 1	1	235	3,579	C++
Max atts.	SPb SU 3	6	266	7,846	C++

Problem D. Distance on Triangulation (1)

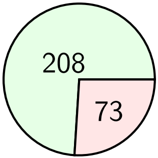
- Divide and conquer; prove that each triangulated polygon has a diagonal that cuts at least $n/3$ vertices
- Randomly picking a diagonal does not work — will time limit
- Recursively split polygons this way for a total depth of $O(\log n)$; get $O(n)$ subpolygons of total size $O(n \log n)$
- Terminal subpolygons for this problem are the ones that do not have have any diagonals to split them further — triangles



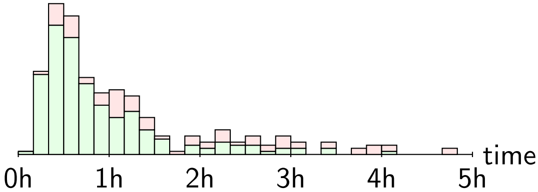
Problem D. Distance on Triangulation (2)

- For each subpolygon precompute the shortest distances from two ends of the diagonals of that was used to cut out this subpolygon from the large one
  - $O(n \log n)$ total memory to store the distances
  - Can be done in $O(n \log n)$ by doing breadth-first search in each subpolygon or in $O(n \log^2 n)$ by doing recursive queries (see below for query implementation)
- Each query can be answered in $O(\log n)$ recursively
  - Terminal subpolygon (triangle) — trivial
  - x and y in query are both on one side of splitting diagonal — recursive query into the corresponding subpolygon
  - x and y in query are on different sides — use precomputed distances to diagonal ends (diagonals do not intersect, so $x - y$ path goes through one of the ends)

Problem E. Easy Problemset



Total



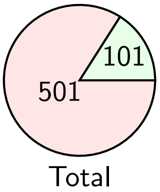
	Java	C++	Total
Accepted	17	191	208
Rejected	9	64	73
Total	26	255	281

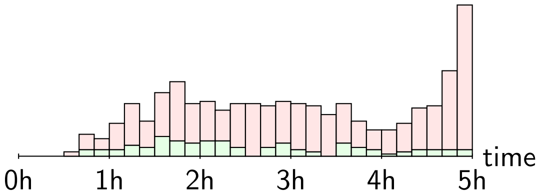
solution	team	att	time	size	lang
Fastest	Ural FU 1	1	8	1,011	C++
Shortest	NU 14	1	22	426	C++
Max atts.	Kyrgyz-Turkish U 1	5	188	781	C++

Problem E. Easy Problemset (1)

- The easiest problem
- Just implement what the problem statement says
- Pay attention to judges without remaining problems — don't forget to propose a hard problem
- Sample inputs and outputs were designed to expose all the tricky cases to make debugging easy

Problem F. Froggy Ford



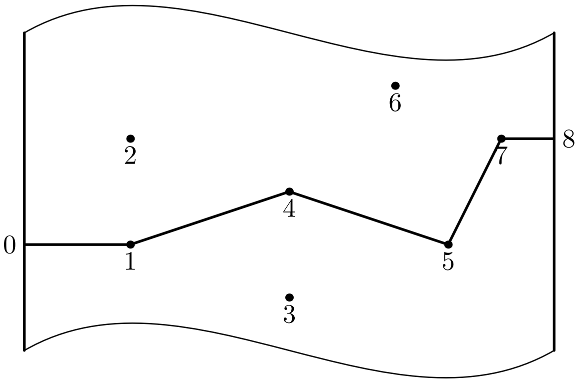


	Java	C++	Total
Accepted	5	96	101
Rejected	12	489	501
Total	17	585	602

solution	team	att	time	size	lang
Fastest	NNSU	1	41	2,281	C++
Shortest	ITMO 4	1	85	1,600	C++
Max atts.	Saratov SU 3	19	217	3,797	C++

Problem F. Froggy Ford (1)

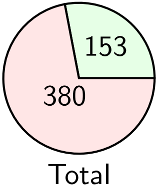
- Consider a graph with vertices $1, \ldots n$ corresponding to stones, 0 for the left shore, $n+1$ for the right one; obvious way to compute distances between vertices
- The problem of finding the optimal route from 0 to $n+1$ as defined in problem is called *minimax path problem*

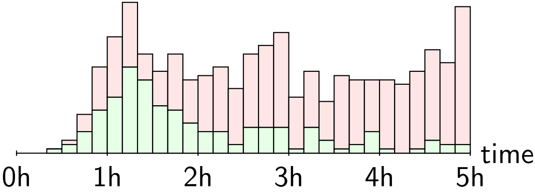


Problem F. Froggy Ford (2)

- Minimax path form 0 to all other vertices can be found by Djikstra algorithm with a corresponding minimax update rule; $O(n^2)$; no need to even have a heap in Djikstra
- Second invocation of the same algo to find minimax path from $n + 1$ to all others
- A new optimal stone can be only at the center between a pair of vertices (stone – stone, stone – shore, shore – shore)
  - Check all pairs of vertices; $O(n^2)$
  - Use precomputed distances to 0 and to $n + 1$ to find distance when new stone is placed; pick the optimal case
  - Make sure to correctly implement distances between shores (vertices 0 and $n + 1$); this case is not covered in sample input

Problem G. Generators





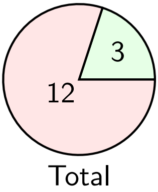
	Java	C++	Total
Accepted	9	144	153
Rejected	53	327	380
Total	62	471	533

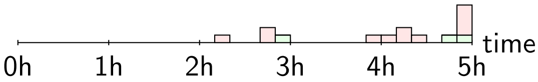
solution	team	att	time	size	lang
Fastest	MSU 3	1	29	3,169	C++
Shortest	Ural FU 4	2	86	1,136	C++
Max atts.	Far Eastern FU	19	262	1,699	C++

Problem G. Generators (1)

- Recap: $x_{i+1}^{(j)} = \left(a^{(j)} x_i^{(j)} + b^{(j)}\right) \mod c^{(j)}$
- Generate $c^{(j)}$ numbers for each LCG — produce all numbers this LCG can possibly generate; for each LCG find:
  - the maximum $x_{t_j}^{(j)}$; pay attention to $x_0^{(j)}$ (don't skip it)
  - the second maximum $x_{u_j}^{(j)}$, such that $\left(x_{t_j}^{(j)} - x_{u_j}^{(j)}\right) \mod k \neq 0$
  - Pay attention to cases when there is no second maximum, e.g. all generated numbers are the same or all differences between them are multiples of k
- When $\sum_{j=1}^{n} x_{t_j}^{(j)} \mod k \neq 0$ — that's the answer
- Otherwise, find j such that $\left(x_{t_j}^{(j)} - x_{u_j}^{(j)}\right)$ is maximized (if at least one second maximum u_j exists) and replace t_j with u_j
- Otherwise, there is no answer

Problem H. Hypercube





	Java	C++	Total
Accepted	0	3	3
Rejected	0	12	12
Total	0	15	15

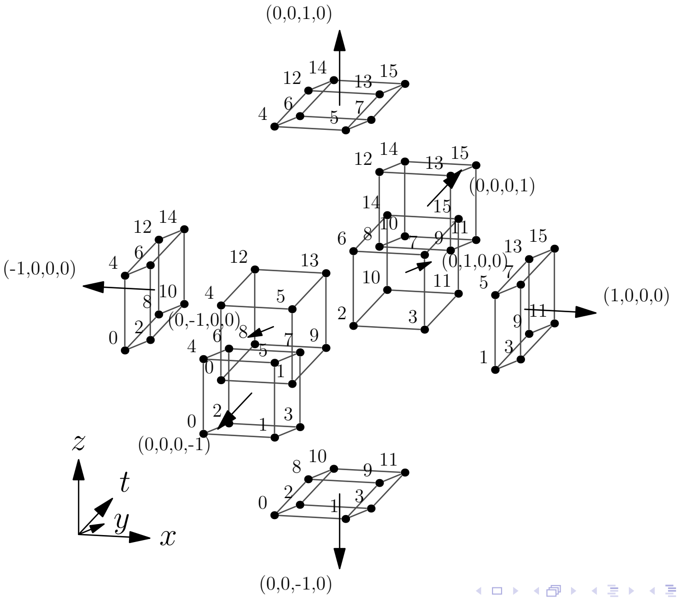
solution	team	att	time	size	lang
Fastest	SPb SU 1	1	175	2,241	C++
Shortest	SPb SU 1	1	175	2,241	C++
Max atts.	Ural FU 1	3	289	5,628	C++

Problem H. Hypercube (1)

- Disassemble tesseract into 8 cubic cells
- Start with an arbitrary cube of an octocube, assume it corresponds to an arbitrary cell of tesseract
- Visit all cubes of a given octocube via DFS
- Each time a cube is visited, see what cell it shall correspond to and if that cell was not used yet
- There are two conceptual ways to uniquely identify tesseract's cells and to traverse them
  - 3D geometry — represent each cell via numbering of its 8 vertices; no 4D vector manipulations required
  - 4D geomerty — represent each cell via a 4D vector normal; leads to simpler code

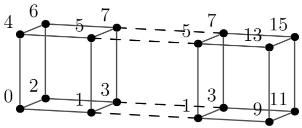
Problem H. Hypercube (2)

- Disassembled tesseract with numbered vertices and normals



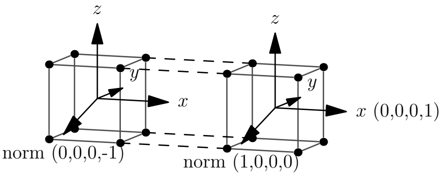
Problem H. Hypercube (3)

- Solution using 3D geometry
  - All vertices of a tesseract are numbered from 0 to 15
  - Cell in a tesseract are represented by cubes of 8 vertex indices
  - Moving in one of 6 directions in an given octocube, find 4 vertices in that direction on a corresponding face of the current cube
  - Among remaining cells find the one that can be rotated/flipped (in 3D) to get a match of vertices face-to-face — this is the next cell and its 3D rot'n/flip

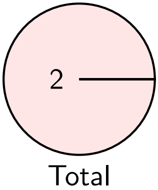


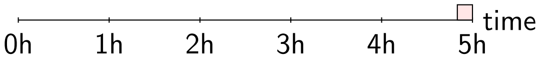
Problem H. Hypercube (4)

- Solution using 4D geometry
  - Keep track of a $4 \times$ 4D vectors: 3 vectors define a basis for current cell's hyperplane, 4th vector defines a normal
  - A normal also uniquely identifies a cell of a tesseract
  - Moving in one of 6 directions in an given octocube, the corresponding basis vector of the current hyperplane (multiplied by ± 1 depending on direction in the axis) becomes the normal of the next cell; the former normal replaces basis vector in that direction (multiplied by ∓ 1) — proper 4D rot'n



Problem I. Iceberg Orders





	Java	C++	Total
□ Accepted	0	0	0
□ Rejected	0	2	2
Total	0	2	2

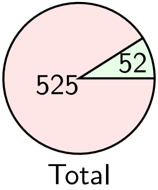
Problem I. Iceberg Orders (1)

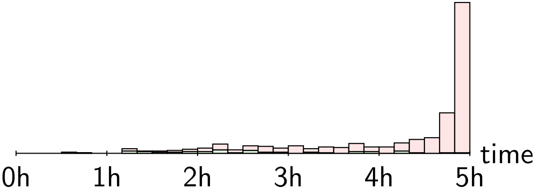
- The structure of the code is hinted in the statement:
  - keep buy and sell orders in the book separately in a sorted tree maps keyed by the price
  - at each price keep a list of orders ordered by priority
  - this way finding a set of orders to match with is efficient — $O(k)$, where k is the number of orders to match with
- The solution is then mostly boils down to implementing what the problem statement says, with one tricky case
- When big order (volume V_a is big) comes in, it produces a lot of trades with other orders that have small tip value TV_b; namely $O(V_a)$ trades — too many to simulate directly
- Recap: V_a is up to 10^9; while the total number of different matched order pairs is guaranteed not to exceed 10^5

Problem I. Iceberg Orders (2)

- The tricky case is addressed by figuring out how many times m an incoming order fully matches with all orders at a current price level
- m is found using binary search in $O(p \log V_a)$ operations, where p is the number of orders at a given price level
- Then, all the m matches can be simulated at one pass in $O(p)$; simulating remaining pass directly
- Care shall taken be in two additional cases
  - when $p \gg k$ make sure that $O(k)$ operations are performed — must do one direct order-by-order match at a given price level first
  - when incoming order volume V_a is so big that the whole price level with k orders is consumed, must do it in $O(k)$; can afford additional log in binary search only at the last matched price level

Problem J. Jump





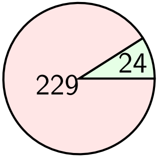
	Java	C++	Total
Accepted	1	51	52
Rejected	43	482	525
Total	44	533	577

solution	team	att	time	size	lang
Fastest	Ural FU 1	1	33	1,401	C++
Shortest	Ural FU 4	5	286	905	C++
Max atts.	Perm SU 1	7	251	1,323	C++

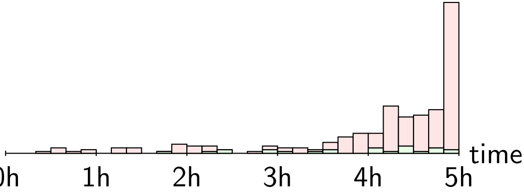
Problem J. Jump (1)

- Recap: must solve in $n + 500$ queries
- Solve the problem in two phases: Phase I with up to 499 queries and Phase II with up to $n + 1$ queries
- Phase I: find Q_{I} such that $\textsc{Jump}(Q_{\mathrm{I}}) = n/2$
  - do random queries in this phase
  - worst case when $n = 1000$, probability of guessing $n/2$ bits in a single random query is $\frac{\binom{1000}{500}}{2^{1000}} = 0.0252\ldots$
  - probability of **not** finding Q_{I} in 499 queries is 2.9×10^{-6}
  - Just quit if $\textsc{Jump}(Q_{\mathrm{I}}) = n$ is found
- Phase II: find solution Q_{II} such that $\textsc{Jump}(Q_{\mathrm{II}}) = n$
  - for $i = 2 \ldots n$ do queries with $Q_i = \{Q_{\mathrm{I}}$ bits 0 and i flipped$\}$
  - $\textsc{Jump}(Q_i) = n/2$ if bits 0 and i has the same "correctness"
  - assume 0 is correct bit in Q_{I}; make $Q_{\mathrm{II}} = \{Q_{\mathrm{I}}$ all bits j' flipped$\}$ where $\textsc{Jump}(Q_{j'}) \neq n/2$; try query Q_{II}; quit if got n
  - assume 0 is not correct; make $Q_{\mathrm{II}} = \{Q_{\mathrm{I}}$ all bits j'' flipped$\}$ where $j'' = 0$ or $\textsc{Jump}(Q_{j''}) = n/2$; must get $\textsc{Jump}(Q_{\mathrm{II}}) = n$

Problem K. King's Inspection



Total

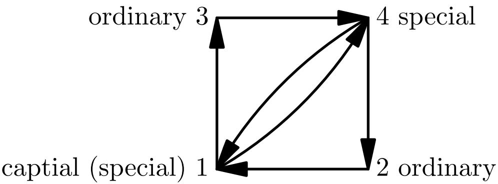


	Java	C++	Total
Accepted	0	24	24
Rejected	6	223	229
Total	6	247	253

solution	team	att	time	size	lang
Fastest	MIPT 5	2	102	2,920	C++
Shortest	ITMO 1	3	212	2,164	C++
Max atts.	NEFU 1	9	279	2,865	C++

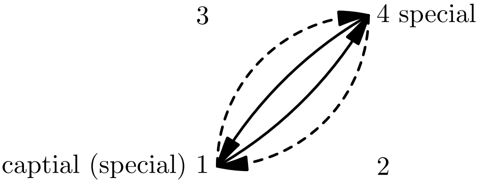
Problem K. King's Inspection (1)

- Count in-degree d_i^{in} and out-degree d_i^{out} for each city i; there is no route if either is zero for any city (important check!)
- Identify *special* cites i: capital ($i = 1$) and cities with $d_i^{\text{in}} > 1$ or $d_i^{\text{out}} > 1$; there are at most 41 special cities
- Other cities are *ordinary*



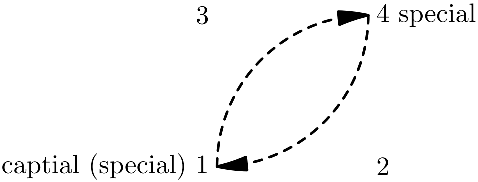
Problem K. King's Inspection (2)

- Merge all paths between special cities though ordinary cities.
- Ordinary cities: non-capital and $d_i^{\mathrm{in}} = 1$ and $d_i^{\mathrm{out}} = 1$
- For each special city create a list of outgoing paths to other special cities
  - there is no route if more than one outgoing path from a special city requires going through ordinary cities
  - if there is one outgoing path through ordinary cities, make it the only path in the outgoing list

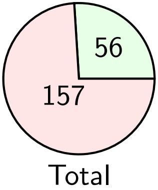


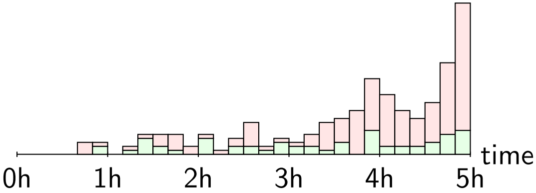
Problem K. King's Inspection (3)

- The picture shows properly reduced graph; but the list of cities on the reduced paths is still kept to print the answer
- Do exhaustive search (backtracking) for a path — at most 2^{20} operations
- There are at most 20 special cities with some choice (more that 1 outgoing path in list)



Problem L. Landscape Improved



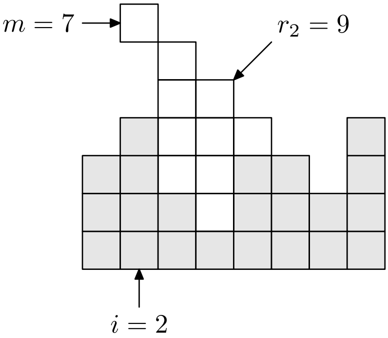


	Java	C++	Total
Accepted	2	54	56
Rejected	8	149	157
Total	10	203	213

solution	team	att	time	size	lang
Fastest	Saratov SU 4	1	53	2,969	C++
Shortest	Kazakh-British TU 2	5	289	1,465	C++
Max atts.	MIPT 3	9	298	2,880	C++

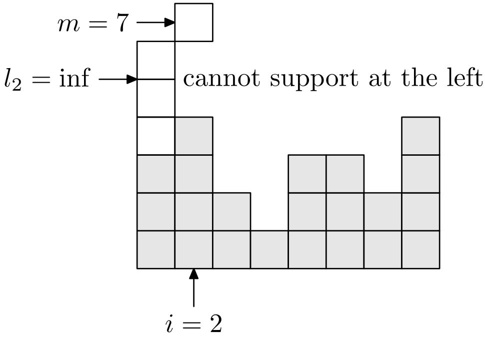
Problem L. Landscape Improved (1)

- Do binary search for the answer; try $O(\log n)$ guesses at the answer in the process
- For each guess m of the answer count the number of squares of stones required to build a mountain of height m, if it is possible; compare the result with n
- Let r_i be the number of squares of stones required to support the mountain of height m with a peak at i at the right



Problem L. Landscape Improved (2)

- Let l_i — the number to support at the left
- Total number of squares $t_i = l_i + r_i + m - h_i$
- The number of required squares is $\min t_i$ for all i
  - r_i is computed with a single pass for i from 1 to w in $O(w)$
  - l_i with a single pass for i from w to 1
  - overall time to find a solution is $O(w \log n)$



Credits

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