Interactive Treasure Hunt

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Let's notice that pairs of points $(x_1, y_1), (x_2, y_2)$ and $(x_1, y_2), (x_2, y_1)$ give the same results for all possible `SCAN` requests. So we will assume that $x_1 \le x_2$ and $y_1 \le y_2$, and then check both these pairs using three `DIG` requests.

First, let's make `SCAN`s in points $(1, 1)$ and $(1, m)$.

$$A = \texttt{SCAN}(1, 1) = (x_1 - 1) + (x_2 - 1) + (y_1 - 1) + (y_2 - 1) \tag{1}$$
$$B = \texttt{SCAN}(1, m) = (x_1 - 1) + (x_2 - 1) + (m - y_1) + (m - y_2) \tag{2}$$

From these values we can find sums:

$$S_x = x_1 + x_2 = \frac{A + B + 6 - 2m}{2} \tag{3}$$
$$S_y = y_1 + y_2 = \frac{A - B + 2 + 2m}{2} \tag{4}$$

Now let's make SCAN in points $(\lfloor \frac{Sx}{2} \rfloor, 1)$ and $(1, \lfloor \frac{Sy}{2} \rfloor)$.

$$C = \texttt{SCAN}(\lfloor \frac{Sx}{2} \rfloor, 1) = (x_2 - x_1) + (y_1 - 1) + (y_2 - 1) \tag{5}$$
$$D = \texttt{SCAN}(1, \lfloor \frac{Sy}{2} \rfloor) = (x_1 - 1) + (x_2 - 1) + (y_2 - y_1) \tag{6}$$

From these values we can find the differences:

$$D_x = x_2 - x_1 = C - S_y + 2 \tag{7}$$
$$D_y = y_2 - y_1 = D - S_x + 2 \tag{8}$$

Now we can find values

$$x_1 = \frac{S_x - D_x}{2} \tag{9}$$
$$x_2 = \frac{S_x + D_x}{2} \tag{10}$$
$$y_1 = \frac{S_y - D_y}{2} \tag{11}$$
$$y_2 = \frac{S_y + D_y}{2} \tag{12}$$

Finally, we can `DIG` in cell (x_1, y_1). If we find the treasure, then the second one must be in cell (x_2, y_2). If not, then the treasures are in the cells (x_1, y_2) and (x_2, y_1).