Global Warming

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Let's do a sweep plane algorithm from top to bottom. We will maintain connected components for points that are higher than the plane and areas of surfaces for these components. When the sweep plane meets a new point, we iterate over its neighbors that are higher than the plane and unite their connected components with the new point. In order to store connected components, you may use DSU.

Now, let's handle areas. Consider a face with vertices a, b and c. If the face is horizontal, we will add its area to the area of a component when the sweep plane reaches it. Otherwise, suppose $z_a \le z_b \le z_c$ and the current high of the sweep plane is s. Then the area of a part of the face that is higher than the sweep plane is:

$$area(s) = \frac{\int_{z=s}^{z_c} l(z)\,dz}{\sin(\alpha)}$$

Where α is an angle between the plane of the face and a horizontal plane. And $l(z)$ is the length of a section of the face by a horizontal plane on high z. Function $l(z)$ is linear on segments $[z_a, z_b]$ and $[z_b, z_c]$. So, function $area(s)$ is a quadratic on the same segments. So, we can maintain the sum of such quadratic functions over all faces of one connected component. And when we meet a new point, we iterate over all faces that contain this point and modify the quadratic function for that face.

Finally, when the sweep plane reaches the high of some query, we look at the connected component of the point in the query.