Adrenaline Rush

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Consider two cars numbered x and y, $x \le y$. Suppose these cars finish at positions i and j, respectively, such that $c_i = x$ and $c_j = y$. There are two possible scenarios:

- $i \ge j$ - The cars finished in the reverse order. For this to occur, car y must overtake car x exactly once, but car x must not overtake car y.
- $i \le j$ - The cars finished in the same order. In this case, either car y overtook car x and then car x overtook car y back, or there were no overtakes between them. To maximize the answer, we assume that two overtakes occurred in this case.

Based on the above, the total number of overtakes does not exceed $n(n-1) - \text{inv}(c)$. Here $\text{inv}(c)$ is the number of pairs (i, j) such that $i \le j$ and $c_i \ge c_j$, called the number of inversions in the permutation c.

To achieve the maximum possible number of overtakes, we can use the following strategy. Consider the cars in reverse order of their finishing positions, i.e., $c_n, \ldots, c_1$. Let each car overtake all the cars in front of it one by one, and then allow all the cars that finished earlier to overtake it.

Applying this algorithm to the first example:

- The cars start in the order of $1, 2, 3$.
- Car 1 finishes last. Since it starts first, there is no other car it can overtake. However, both car 2 and car 3 should overtake it. The relative order at this point is $2, 3, 1$, with a total of 2 overtakes so far.
- Car 3 finishes second. It can overtake car 2 in front of it, after which car 2 overtakes it back. This results in the same relative order $2, 3, 1$, with a total of 4 overtakes.
- Car 2 finishes first, but there is no car it can overtake at this point.
- In total, this gives a sequence of 4 overtakes.