Managing Cluster

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Despite this problem is formulated as a maximization problem, this is a constructive problem.

Let us find a maximum matching in this tree. It can be done with dp or a greedy algorithm in linear time. Let us define a set of edges in this matching as $M = \{(s_i, f_i)\}_{i=1}^{k}$.

Obviously, the answer does not exceed $|M|$. Let us construct swaps, such that the answer is exactly $|M|$ and equal replicas are placed in edges from M.

Let us construct a graph on n services, where we will connect services a_{s_i}, a_{f_i} with an edge. Each service will have a degree of at most 2, so this graph is a union of paths and cycles.

Let us consider some cycle. Suppose it consists of c edges $(p_1, p_2), (p_3, p_4), \ldots, (p_{2c-1}, p_{2c})$, where p_i are machines and $a_{p_{2i}} = a_{p_{2i+1}}$. Let us make $c - 1$ swaps between machines: $p_1 \leftrightarrow p_{2c-1}, p_2 \leftrightarrow p_{2c-2}, \ldots, p_{c-1} \leftrightarrow p_{c+1}$. As a result, all edges in the cycle will have replicas of equal services.

To solve paths, let us do the same operation as with cycles. We can do that because we can just consider that the start and the end of the path are connected.

With the constructed swaps, we will have the optimal answer. Time complexity $O(n)$.