Knowns and Unknowns

Problem author: Tikhon Evteev; problem developer: Niyaz Nigmatullin

Get some professor's list, it consists of permutation elements and wildcards. Define a set of blocks consisting of:

1. a known element of the list;
2. a non-extendable set of consecutive d unknown elements.

For each second type of blocks there is a set of elements such that any of its subset of size d can be there, call the elements of this set as *candidates* for this block. This set consists of some elements from original permutation that are between the element on the left, and the element on the right of this block. For each first type of blocks the only candidate is the one that is known at that position. Note that each element is a candidate to at most one of the blocks.

Let's build the blocks for both professors.

Then we can build a flow network similar to bipartite matching network, the left part is for the blocks of the first professor, and the right — of the second one. From the source to each block of the first professor we have edges with capacity "the size of the block". The same is for the blocks of the second professor and sink. For each integer from 1 to n, if it is a candidate to a block of first professor, and to a block of second professor, then add an edge with capacity 1 from the first block to the second.

If the maximum flow in the network is less than k, then the data is inconsistent.

Otherwise, for each edge we need to check "is there a maximum flow with this edge in it?", and "is there a maximum flow without this edge in it?". Find a maximum flow, if an edge is in the maximum flow, then check if there is a cycle in a residual network with the reverse of the edge. If an edge is not in the maximum flow, then check if there is a cycle in a residual network with the edge. This is done by finding a path between the end of the edge and its beginning in the residual network, the path wouldn't contain the reverse edge, since it is already saturated. The solution works in $O(n^2)$.